New integrated approach to the problem of ranking and supplier selection under uncertainties

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Abstract: The problem of ranking and supplier selection in the uncertain environment is part of a purchasing plan and its relationship has a critical effect on the competitive advantage (high-quality products at lower cost with higher customer satisfaction) of each industrial organization. The considered problem can be stated as a multicriteria decision problem which includes both quantitative and qualitative criteria. The criteria present supplier performances which are defined by the purchasing Management Team depending on the size of industrial organizations and on production type. In this paper, the Management Team, using European Union (EU) recommendations, made a choice of criteria for supplier evaluation. The fuzzy rating weights of each pair of the considered criteria and uncertain criteria values are described by linguistic expressions which are modelled by triangular fuzzy numbers. The fuzzy extent approach for the synthetic extent values of the pairwise comparison for handling fuzzy analytic hierarchical process (AHP) is used to calculate the weight vector. The extension of the fuzzy technique for order preference by similarity to ideal solution (TOPSIS) is applied to rank the suppliers. The proposed model is illustrated by an example. It is shown that the developed model is highly suitable as a decision-making tool for reaching decisions about supplier selection.

Keywords: supplier selection, multicriteria analysis, AHP, TOPSIS

1 INTRODUCTION

Technological, political, economic, and environmental changes in the business world require managers of industrial organizations to develop new strategies which should lead to an increase in their competitive advantages (high-quality products at lower cost with higher customer satisfaction). One of these strategies implies developing purchasing management of raw materials and components (materials). According to reference [**1**] supplier selection is one of the most important functions of purchasing management.

Based on theory and the practice of purchasing management, it is well known that selection of the best supplier from the group of possible suppliers is always based on multicriteria whose objectives are conflicting; thus the selection of the appropriate suppliers is far from a trivial task. The common criteria regarding which best supplier is selected are unit cost and quality of material. If the supplier selection problem is related to the global sourcing, then it is necessary to consider many other criteria such as: political-economic situation, geographical location, infrastructure, financial background, performance history, risk factors, etc. The number and type of criteria according to which best supplier is selected are determined by purchasing managers and multiple criteria need to be carefully examined. The problem becomes significantly more complex when introducing the assumption, which is realistic one, that the considered criteria have different relative importance.

When companies outsource a significant part of their business and become more dependent on outsourcers, the company performance concerning quality and delivery totally depends on its outsources. The consequences of poor decision making

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become more severe. It is, therefore, very important for an outsourcing type of manufacturer to evaluate, manage, and select their suppliers.

The problem of the selection of the best supplier can be characterized as dynamic and unstructured. Essentially, the considered problem is a group-making problem under multiple criteria. The environment changes rapidly or becomes uncertain, thus making the values of some criteria difficult or impossible to quantify. In this case, the values of uncertain criteria can be adequately described by linguistic expressions which are modelled by applying the fuzzy set theory [**2–4**].

The problem of ranking and supplier selection is a group decision-making problem under multiple criteria, and it can be described as a multicriteria optimization problem (MADM). In solving it, three parameters should be taken into account – the degree of uncertainty, the number of decision makers, and the nature of the criteria. Different decision criteria may vary depending on the needs of the organization and changes in the environment. The estimation of criteria weights and uncertain criteria values cannot be preformed with an exact numerical value. It seems a more realistic approach to use linguistic assessments instead of numerical values. In other words, all uncertainties existing in the considered problem can be described by linguistic variables [**2**]. In this paper, modelling of these linguistic variables is based on the fuzzy set theory [**2–4**].

The fuzzy set theory can provide a valuable tool which copes with three major problematic areas of supplier selection: imprecision, randomness, and ambiguity. As far as imprecision is concerned, it provides a powerful tool to assess selection criteria importance. As far as ambiguity is concerned, it copes better than other methods with the treatment of linguistic variables. Fuzzy logic makes it possible to emulate the human reasoning process and make a decision based on vague or imprecise data [**5**].

The fuzzy set theory resembles human reasoning in its use of approximate information and uncertainty to generate decisions. It can be demonstrated that the fuzzy approach has numerous advantages in treating uncertainties in real-word applications when compared with other approaches such as applying probability theory, applying rough set theory, etc. The advantages of fuzzy system models are outlined as follows [**6**]: conceptually easy to understand; flexible; capture most non-linear functions of arbitrary complexity; tolerant of imprecise data; built on the expertise of experts; can be blended with conventional control techniques; are based on natural languages; and provide better communication between experts and managers.

The rank of suppliers can be obtained either by applying some developed MADM techniques or by combining different MADM techniques. The analytic hierarchical process (AHP) method and the technique for order preference by similarity to ideal solution (TOPSIS) method or the combination of the two has the widest application in the multicriteria supplier selection problem [**7**]. In the AHP method [**8**] the decision-making problem is hierarchally structured: the weight criteria and preference of alternatives under each treated criterion are assigned according to pairwise comparison matrix of the considered factors. The elements of these matrices are obtained upon the evaluation of decision makers. These matrices represent input data for ranking and best alternative selection with respect to all treated criteria and their weights. TOPSIS [**9**] is based on the best alternative selection, which has the shortest distance from the positive-ideal alternative and the longest distance from the negative-ideal alternative. In the conventional forms of the AHP and TOPSIS methods, only crisp parameter values have been considered for supplier selection, which represents their basic deficiency.

In many papers in the literature, the considered problem is solved by the proposed two-stage method. At the first stage, either the AHP or FAHP (fuzzy analytic hierarchical process) method is used to determine the weight of treated criteria and/or the weight of suppliers [**10–13**]. At the second stage, some other methods are used in order to determine the best supplier with respect to all treated criteria, simultaneously, as well as their relative importance. A brief literary review of papers dealing with supplier evaluation and selection by fuzzy AHP and fuzzy TOPSIS approaches is given in this paper.

In reference [**13**], the supplier selection problem was solved by integrating fuzzy AHP and fuzzy TOPSIS. The developed methodology consists of two steps: in the first step, FAHP is applied to determine the relative weights of the evaluation criteria. The authors developed a procedure for determining criteria weights and defined the pairwise comparison matrix according to conventional AHP method. After that, they transformed the real elements of this pairwise comparison matrix into triangular fuzzy numbers. In the following step, before conducting all the calculations of the vector of the priorities, these fuzzy elements had to be normalized by applying a simple normalization procedure. The criteria weights are calculated as the average values of elements of each row from the matrix which is obtained in the previous step. In the second step, the fuzzy TOPSIS method is applied to rank the alternatives.

In reference [**10**], a fuzzy decision-making approach to deal with the supplier selection problem in the supply chain is presented. Weights of all criteria and the rating of each alternative with respect to each criterion are described by linguistic variables which are modelled by trapezoidal or triangular fuzzy numbers. In this way, the decision matrix is converted into a fuzzy decision matrix. According to the concept of the TOPSIS method, the authors defined the fuzzy positive-ideal solution (FPIS) and fuzzy negative-ideal solution (FNIS). The distance of each alternative from FPIS and FNIS is calculated by applying the vertex method. A closeness coefficient of each alternative is defined to determine the ranking order of all alternatives.

In this paper, a pairwise comparison matrix of relative importance was constructed for the considered criteria. The elements of this matrix are defined as the relative importance of criterion k over criterion k' $(k,k'=1,\ldots,K; k\neq k')$, with analogy to references [**11**] and [**13**]. A certain number of decision makers uses linguistic expressions to describe the relative importance of each pair of treated criteria. The fuzzy rating of each decision maker is modelled by triangular fuzzy numbers. The weight vector is calculated by using the extent analysis method [**14**].

The criteria values can be crisp and uncertain. Normalization of crisp criteria is performed according to the normalization procedure which is defined in the conventional TOPSIS method. Uncertain criteria values are described by triangular fuzzy numbers. The domains of these triangular fuzzy numbers belong to the interval [0–1], so that there is no need to apply normalization procedure to uncertain criteria, which significantly decreases the amount of calculation.

A closeness coefficient according to which rank of suppliers is determined is calculated using the procedure defined in the conventional TOPSIS method. The value of the closeness coefficient for each supplier is described by a fuzzy number according to fuzzy algebra rules [**3**]. The rank of suppliers is determined with respect to closeness coefficients. The comparison of determined closeness coefficients is based on the method for comparison of fuzzy numbers (see Appendix 3).

When comparing papers which propose a model for ranking suppliers under uncertainties, certain differences could be noted, which are further described below. This analysis also shows the advantages of the proposed model.

In papers [**10**, **13**] and [**15**] to [**17**], criteria weights are described by linguistic expressions which are modelled by trapezoidal fuzzy numbers [**10**] or triangular fuzzy numbers [**15–17**]. It appears that the weight determination of criteria is more reliable when obtained using a pairwise comparison than when they are directly obtained, because it is easier to make a comparison between two criteria than to make an overall weight assignment. In references [**11**] and [**13**] the fuzzy evaluation matrix of the criteria is performed by one decision maker. The current authors are of the opinion that the estimation is more precise if the problem is posed as a multi-criteria group decision-making problem, as in the present paper. Also, in reference [**13**] values of criteria weights are calculated as the average of the elements of each row from a pairwise comparison matrix of the relative importance of the criteria. In this case, criteria weights are described by fuzzy numbers as in references [**15**] to [**17**], which complicates the process of calculation, but does not contribute to obtaining more precise results.

In references [**10**, **11**, **13**] and [**15**] to [**17**] it is supposed that all considered criteria are uncertain. In practice, such a supposition cannot be taken as a completely suitable one, as values of many criteria cannot be determined with absolute precision (e.g. distance from a supplier). In this paper, an effort is given to observe simultaneously both crisp and uncertain criteria in the problem of supplier ranking.

In references [**10**, **11**, **13**] and [**15**] to [**17**], fuzzy positive and fuzzy negative ideal solutions are determined according to the weighted normalized fuzzy decision matrix by using different methods. In the proposed model, positive-ideal and negative-ideal solutions for each criterion are determined according to the normalized decision matrix with respect to the type of criterion. The distance from both the positive ideal and negative ideal solution is calculated with respect to expressions from conventional TOPSIS.

The paper is organized as follows. A multicriteria approach to the problem of ranking and selecting the best supplier is given in section 2. In section 3, a synthetic extent analyses method for calculating the final priority weights of criteria is presented. Modelling of uncertain criteria values is described in section 4. In section 5, principles of fuzzification of the TOPSIS method are presented, whereas section 6 gives an illustrative example.

2 PROBLEM STATEMENT: BASIC ASSUMPTIONS

The problem of ranking and supplier selection is, in fact, a group multiple-criteria decision-making problem (GMCDM). The following are assumptions underlying a model of the considered problem.

1. Suppliers obtaining only one kind of raw material that is, at the same time, which is most important in the sense of maintaining the continuity of the production process in industrial organization, are considered.
2. Management team (production managers, supply managers and marketing managers) defines the group of possible suppliers. The selection of suppliers is based on: (a) analysis of historical

data based on the experience of other companies, (b) using data which are found in official bulletins, (c) judgements of experts, (d) professional observation, etc. In practice, different approaches are more often combined.
3. Management team defines the group of criteria according to which each possible supplier is evaluated. The number and types of these criteria depend on the size of industrial organization, type of industries, etc. The problem of selection of the criteria according to which suppliers are evaluated and selected can be observed as an isolated problem. In the literature, there are numerous papers emphasizing decision criteria that should be used. The classification given by the European managers is adopted in this paper.
4. To each defined criterion an organized pair (relative importance, value joined) is associated.
5. Relative importance of treated criteria does not depend on suppliers and is in most cases hardly changed. Generally, the relative importance of criteria is different and determined according to knowledge and experience of the management team, and it can be stated as follows: values of defined criteria are determined for each supplier separately. In the considered problem, these values can be crisp and/or uncertain. The uncertain values are modelled by applying the fuzzy set theory.

3 MODELLING OF CRITERIA WEIGHTS

The management team determines the number and kind of criteria primarily depending on the type of industry and size of considered industrial organization. Ordered pair, importance, and value is associated with each treated criterion.

All the criteria for evaluating suppliers are usually not of the same relative importance, and do not depend on the supplier. Also, they can be considered as unchangeable during the considered period of time. They involve a high degree of subjective judgement and individual preferences of decision makers. It is thought that the judgement of each pair of treated criteria best suits human-decision nature (by analogy with the AHP method). In conventional AHP, the pairwise comparison is established using a standard integer scale [1–9]. Value 1 denotes that criterion k is as important as criterion k', and value 9 denotes that criterion k is much more important than criterion $k', k, k' = 1, \ldots, K; k \neq k'$.

The use of discrete scale of AHP is simple and easy, but it is not sufficient to take into account the uncertainty associated with the mapping of one's perception to a number [**18**]. Decision makers express their judgements far better by using linguistic expressions than by representing them in terms of precise numbers. It feels more confident to give interval judgements than fixed value judgements.

In this paper, the fuzzy rating of each decision maker is described by linguistic expressions which can be represented as triangular fuzzy number $\tilde{W}^e_{kk'} = \left(x; l^e_{kk'},\ m^e_{kk'},\ u^e_{kk'}\right)$ with the lower and upper bounds $l^e_{kk'},\ u^e_{kk'}$ and modal value $m^e_{kk'}$, respectively. The greater $u^e_{kk'} - l^e_{kk'}$, the fuzzier the degree. Values in the domain of these triangular fuzzy numbers belong to a real set within the interval [1–9]. Values in the domain of each of these five fuzzy numbers have the same meaning as the value of a standard scale which is given in conventional AHP.

If the strong relative importance of criterion k' over criterion k holds, then the pairwise comparison scale can be represented by the fuzzy number

$$\tilde{W}_{kk'} = \left(\tilde{W}_{k'k}\right)^{-1} = \left(\frac{1}{u_{k'k}}, \frac{1}{m_{k'k}}, \frac{1}{l_{k'k}}\right)$$

If $k = k'$ $(k, k, = 1, \ldots, K)$ then the relative importance of criterion k over criterion k' is represented by single point 1 which is a triangular fuzzy number (1,1,1).

The aggregated fuzzy rating of relative importance of each pair of the considered criteria must include the fuzzy rating of all decision makers. The aggregated fuzzy rating can be defined as

$$l_{kk'} = \min_{e=1,\ldots,E} l^e_{kk'} \quad m_{kk'} = \frac{1}{E}\sum_{e=1}^{E} m^e_{kk'} \quad u_{kk'} = \max_{e=1,\ldots,E} u^e_{kk'}$$

The relative importance of each pair of the considered criteria is described by the triangular fuzzy number $\tilde{W}_{kk'} = (l_{kk'},\ m_{kk'},\ u_{kk'})$ with the lower and upper bounds $l_{kk'}, u_{kk'}$ and modal value $m_{kk'}$, respectively.

In this paper, the fuzzy rating of each decision maker can be described by using five linguistic expressions: equally important, moderately important, strongly important, very strongly important, and most important. These linguistic expressions are modelled by triangular fuzzy numbers which are given in the following way:

(a) Equally important $\tilde{R}_{\mathrm{E}} = (x;\ 1,\ 1,\ 2)$
(b) Moderately important $\tilde{R}_{\mathrm{M}} = (x;\ 1.5,\ 3,\ 4.5)$
(c) Strongly important $\tilde{V}_{\mathrm{S}} = (x;\ 3.5,\ 5,\ 6.5)$
(d) Very strongly important $\tilde{R}_{\mathrm{VS}} = (x;\ 6,\ 7,\ 8)$
(e) Most important $\tilde{R}_{\mathrm{VVS}} = (x;\ 7.5,\ 9,\ 9)$

These triangular fuzzy numbers, as the simplest shape of membership functions are shown in Fig. 1.

The domain of each used fuzzy number is defined on a set of real numbers belonging to the interval [1–9], with analogy to Saaty's scale of measures [**8**]. Values of the domain are represented on the x-axis,

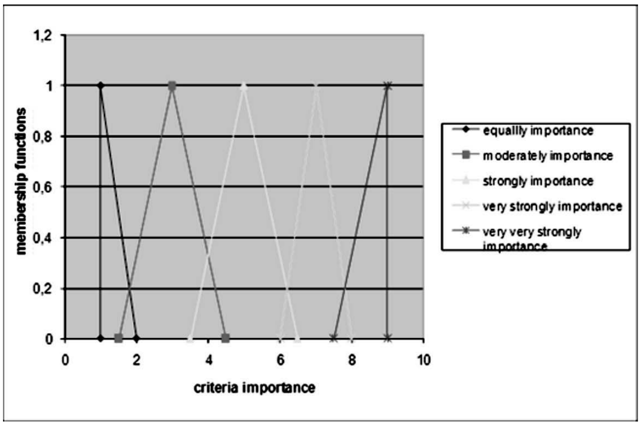

Fig. 1 Five triangular fuzzy numbers that describe the relative importance of the criteria

whereas membership function values of each used fuzzy number are presented on the y-axis.

3.1 Calculating of criteria priority weights

The weight vector of the considered criteria is calculated by applying the concept of extent analysis [**18**].

Let $X = \{x_1, ..., x_i, ..., x_K\}$ be an object set, and $Y = \{y_1, ..., y_j, ..., y_K\}$ be a goal set. According to the concept of extent analysis [**14**], each object is taken and the extent analysis for each goal is performed, respectively. Therefore the K extent analysis values for each object are marked by the following signs

$$N_i^1, \quad , N_i^j, ..., N_i^K, \ (i = 1, \ldots K)$$

where

N_i^j are triangular fuzzy numbers ($j = 1,...,K$).

The value of fuzzy synthetic extent with respect to the ith object is defined as

$$\tilde{S}_i = \sum_{j=1}^{K} N_i^j \cdot \left[\sum_{i=1}^{K}\sum_{j=1}^{K} N_i^j\right]^{-1}$$

where

$$\sum_{j=1}^{K} N_i^j = \left(\sum_{k'=1}^{K} l_{kk'}, \sum_{k'=1}^{K} m_{kk'}, \sum_{k'=1}^{K} u_{kk'}\right)$$

$$\sum_{i=1}^{K}\sum_{j=1}^{K} N_i^j = \left(\sum_{k=1}^{K}\sum_{k'=1}^{K} l_{kk'}, \sum_{k=1}^{K}\sum_{k'=1}^{K} m_{kk'}, \sum_{k=1}^{K}\sum_{k'=1}^{K} u_{kk'}, \right)$$

$$\left[\sum_{i=1}^{K}\sum_{j=1}^{K} N_i^j\right]^{-1} = \left(\frac{1}{\sum_{k=1}^{K}\sum_{k'=1}^{K} u_{kk'}}, \frac{1}{\sum_{k=1}^{K}\sum_{k'=1}^{K} m_{kk'}}, \frac{1}{\sum_{k=1}^{K}\sum_{k'=1}^{K} l_{kk'}}\right)$$

The weight vector is represented as

$$W_p = \left(\left(Bel\left(\tilde{S}_1\right)\right), ..., \left(Bel\left(\tilde{S}_i\right)\right), ..., \left(Bel\left(\tilde{S}_K\right)\right)\right)$$

where $Bel\left(\tilde{S}_i\right)$ is a measure of belief according to which triangular fuzzy number $\tilde{S}_i$ is bigger than all other triangular fuzzy numbers $\tilde{S}'_i$, $(i, i' = 1, \ldots, K; i \neq i')$. This value is obtained by applying the method for fuzzy number comparison (see Appendix 3).

After normalizing W_p, the normalized weights vector W is obtained

$$W = (w_1, ..., w_k, ..., w_K)$$

W is a non-fuzzy number and this gives the priority weights of one criterion over the other.

4 MODELLING OF UNCERTAIN CRITERIA

Values of most criteria, such as quality of material, flexibility of delivery, etc., cannot be stated quantitatively, because managers most often base their estimates on evidence data. In these cases, their values are adequately described by linguistic expressions. In this paper these linguistic expressions are modelled by triangular fuzzy numbers $\tilde{v}_{sk} = (y; l_{sk}, m_{sk}, u_{sk})$ with the lower and upper bounds l_{sk} and u_{sk} and modal value m_{sk}, respectively. The greater u_{sk}-l_{sk}, the fuzzier the degree (see Appendix 2). Values in the domain of these triangular fuzzy numbers belong to real set within the interval [0,1]. Value 0 stands for the lowest criterion value and value 1 for the highest criterion value.

The number and kinds of linguistic expressions are defined by management team depending on the number of uncertain criteria and the number of suppliers and estimate of managers. For the problem considered in this paper, it is realistic to assume the following linguistic expressions which are used to describe the values of uncertain criteria: very high value, high value, moderate value, low value, and very low value.

Triangular fuzzy numbers describing the values of uncertain criteria are given in the following way:

(a) Very low value $\tilde{V}_{VL} = (x; 0, 0.1, 0.25)$
(b) Low value $\tilde{V}_L = (x; 0.15, 0.3, 0.4)$
(c) Moderate value $\tilde{V}_M = (x; 0.35, 0.5, 0.65)$
(d) High value $\tilde{V}_H = (x; 0.55, 0.7, 0.85)$
(e) Very high value $\tilde{V}_{VH} = (x; 0.75, 0.9, 1)$

The domain of each fuzzy number describing uncertain criteria values is defined on the set of real numbers in the interval [0–1]. The domain values of each used fuzzy number are represented on the x-axis, Fig. 2. Membership function values are represented on the y-axis, Fig. 2.

As the domain of each fuzzy number describing uncertain criteria values is defined on the interval [0–1], there is no need to perform uncertain criteria

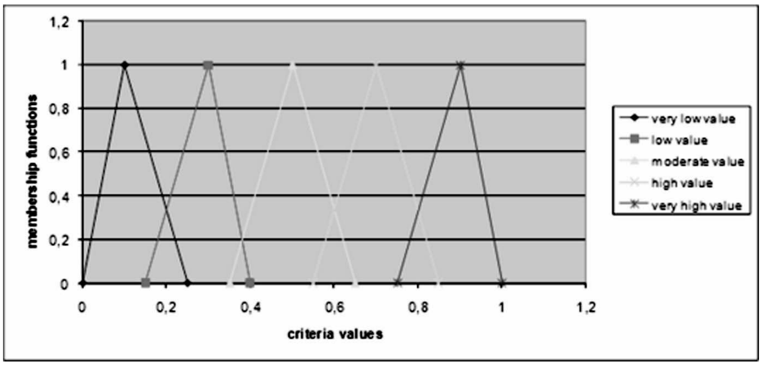

Fig. 2 Five triangular fuzzy numbers that describe values of uncertain criteria

value normalization. In this way the amount of calculation is reduced, while the precision of the achieved result remains the same.

5 PRINCIPLES OF FUZZY TOPSIS METHOD

The TOPSIS method is based on choosing the best alternative, which has the shortest distance from the positive-ideal solution, and the longest distance from the negative-ideal solution [**9**].

In this section, a systematic approach to extend TOPSIS is proposed to solve the supplier selection problem under uncertainties.

The algorithm of the proposed method is realized in the following steps:

Step 1: Calculation of weight vector of the considered criteria by applying procedure which is presented in section 3.

Step 2: Calculation of normalized values for crisp criteria

$$f_{sk}^{n} = \frac{f_{sk}}{\sum_{s=1}^{S} f_{sk}} \quad (1)$$

Step 3: Determining the positive-ideal solution, v_k^+, and the negative-ideal solution, v_k^-, of all crisp criteria:

(a) for a benefit type criterion k, $k = 1,\ldots,K'$

$$v_k^+ = \max_{s=1,\ldots,S} f_{sk}^n,\ v_k^- = \min_{s=1,\ldots,S} f_{sk}^n \quad (2)$$

(b) for a cost-type criterion k, $k = 1,\ldots,K'$

$$v_k^+ = \min_{s=1,\ldots,S} f_{sk}^n,\ v_k^- = \max_{s=1,\ldots,S} f_{sk}^n \quad (3)$$

Step 4: Determining the positive-ideal, $\tilde{v}_k^+$ and negative-ideal solution, $\tilde{v}_k^-$, of all uncertain criteria:

(a) for a benefit type criterion k, $k = K'+1,\ldots,K$

$$\tilde{v}_k^+ = \max_{s=1,\ldots,S} \tilde{v}_{sk},\ \tilde{v}_k^+ = \max_{s=1,\ldots,S} \tilde{v}_{sk} \quad (4)$$

(b) for a cost-type criterion k, $k = K'+1,\ldots,K$

$$\tilde{v}_k^+ = \min_{s=1,\ldots,S} \tilde{v}_{sk} + \tilde{v}_k^+ = \max_{s=1,\ldots,S} \tilde{v}_{sk} \quad (5)$$

The values $\tilde{v}_k^+$ and $\tilde{v}_k^-$, are determined by using comparison method of fuzzy numbers (see Appendix 3).

Step 5:. The distance of each supplier s, $s = 1,\ldots,S$, from the positive-ideal solution, $\tilde{d}_s^M$, and the negative-ideal solution, $\tilde{d}_s^m$, are calculated

$$\tilde{d}_s^M = \sum_{k=1}^{K'} w_k \cdot \left|v_k^+ - f_{sk}^n\right| + \sum_{k=K'+1}^{K} w_k \cdot \left|\tilde{v}_k^+ - \tilde{v}_{sk}\right| \quad (6)$$

$$\tilde{d}_s^m = \sum_{k=1}^{K'} w_k \cdot \left|v_k^- - f_{sk}^n\right| + \sum_{k=K'+1}^{K} w_k \cdot \left|\tilde{v}_k^- - \tilde{v}_{sk}\right| \quad (7)$$

Values which are determined by applying expressions (6) and (7) are described by a triangular fuzzy number [**3, 4**].

Step 6: A closeness coefficient, $\tilde{c}_s$ $(s = 1,..,S)$ is obtained as

$$\tilde{c}_s = \frac{\tilde{d}_s^m}{\tilde{d}_s^m + \tilde{d}_s^M} \quad (8)$$

These values are described by fuzzy numbers according to rules of fuzzy algebra [**3, 4**]. The modal value of fuzzy number $\tilde{c}_s$ is denoted as m_s $(s = 1,\ldots,S)$.

Step 7: Rank all $\tilde{c}_s$ with decreasing order of m_s $(s = 1,\ldots,S)$

6 ILLUSTRATIVE EXAMPLE

A manufacturing company wants to select a suitable material supplier to supply the key components of

Table 1 Values of the considered criteria for each supplier

	$k=1$	$k=2$	$k=3$	$k=4$	$k=5$	$k=6$	$k=7$
$s=1$	285	195	7	Very low value	Very high value	Moderate value	High value
$s=2$	100	200	1	Low value	High value	Very high value	Moderate value
$s=3$	184	155	21	Moderate value	Very low value	High value	Very high value
$s=4$	60	174	15	High value	Low value	Very high value	Very low value
$s=5$	400	165	30	Very high value	Moderate value	Very low value	Low value
$s=6$	332	120	45	Very low value	Moderate value	High value	Moderate value
$s=7$	55	200	1	Low value	Very high value	Moderate value	Very low value
$s=8$	420	106	7	Moderate value	High value	Low value	High value
$s=9$	205	182	15	High value	Low value	High value	Very high value
$s=10$	125	111	7	Very high value	Very low value	Moderate value	Moderate value

products. After preliminary screening, ten possible suppliers remain for further evaluation. A committee of three decision-makers has been formed to determine the criteria weights. Seven criteria, defined according to prominent researches in this area carried out in Europe in 2003, are considered:

(c) $k=1$, distance of supplier, in km
(d) $k=2$, unit price, in euro/product
(e) $k=3$, lead time, in days,
(f) $k=4$, quality of materials
(g) $k=5$, delivery flexibility
(h) $k=6$, financial stability and strength (financial background)
(i) $k=7$, integrity and communication frequency

Table 1 shows values of each observed criterion for each supplier. These values are input data. The first three criteria values for each supplier are crisp and they are based on the evidence data. Values of the other observed criteria are uncertain and they are based on expert evaluation. In other words, a linguistic value, i.e. one of five defined linguistic expressions, is assigned to each uncertain criterion.

Step 1: Three decision-makers use the linguistic weighting variables shown in Fig. 1 to assess the relative importance of each pair of the considered criteria.

The elative importance of the criteria is given in the shape of a matrix, Table 2. Three linguistic values of the relative importance relation are assigned to each pair of the observed criteria. Each one of the three experts of the management team can use five linguistic expressions previously defined to determine the relative importance relation of each pair of the observed criteria. When the criterion k' is less important than k'', $k',k''=1,\ldots,K; k'\neq k''$, it is represented by inverse proportion, with analogy to Saaty's scale of measures [**8**], which is presented by $(\tilde{R})^{-1}$ in Table 2.

Table 3 shows the fuzzy rate of each pair of the observed criteria based on the evaluation of all three experts in the management team.

By applying the concept of extent analysis, the value of fuzzy synthetic extent with respect to the ith criterion, $\tilde{S}_i$, is calculated

$$\tilde{S}_1=(8.03,\ 15.8,\ 25.3)\cdot\left(\frac{1}{158.9},\frac{1}{98.7},\frac{1}{42.63}\right)=(0.05,\ 0.16,\ 0.59)$$

$$\tilde{S}_2=(10,\ 22.3,\ 34.5)\cdot\left(\frac{1}{158.9},\frac{1}{98.7},\frac{1}{42.63}\right)=(0.06,\ 0.23,\ 0.81)$$

$$\tilde{S}_3=(4.2,\ 9.7,\ 18)\cdot\left(\frac{1}{158.9},\frac{1}{98.7},\frac{1}{42.63}\right)=(0.03,\ 0.1,\ 0.42)$$

$$\tilde{S}_4=(6.4,\ 17.6,\ 28.5)\cdot\left(\frac{1}{158.9},\frac{1}{98.7},\frac{1}{42.63}\right)=(0.04,\ 0.18,\ 0.67)$$

$$\tilde{S}_5=(3.2,\ 6.7,\ 11.2)\cdot\left(\frac{1}{158.9},\frac{1}{98.7},\frac{1}{42.63}\right)=(0.02,\ 0.07,\ 0.26)$$

$$\tilde{S}_6=(9,\ 24.3,\ 37.5)\cdot\left(\frac{1}{158.9},\frac{1}{98.7},\frac{1}{42.63}\right)=(0.06,\ 0.25,\ 0.88)$$

$$\tilde{S}_7=(1.8,\ 2.3,\ 3.9)\cdot\left(\frac{1}{158.9},\frac{1}{98.7},\frac{1}{42.63}\right)=(0.01,\ 0.02,\ 0.09)$$

The degree of belief that a fuzzy number $\tilde{S}_i$ is bigger than/equal to other fuzzy numbers $\tilde{S}_{i'}$ $(i,i'=1,..,K;\ i\neq i')$ (see Appendix 3).

$$Bel\left(\tilde{S}_1\geqslant\left(\tilde{S}_2,\ldots.,\tilde{S}_7\right)\right)=\min(0.89,\ 1,\ 0.98,\ 1,\ 0.86,\ 1)=0.86$$

$$Bel\left(\tilde{S}_2\geqslant\left(\tilde{S}_1,\tilde{S}_3,\ldots.,\tilde{S}_7\right)\right)=\min(1,\ 1,\ ,\ 1,\ 0.97,\ 11)=0.97$$

Table 2 Relative importance of each pair of the considered criteria

	$k=1$	$k=2$	$k=3$	$k=4$	$k=5$	$k=6$	$k=7$
$k=1$	(1.1.1)	$(\tilde{R}_M)^{-1}$ $\tilde{R}_E$ $(\tilde{R}_M)^{-1}$	$\tilde{R}_M$ $\tilde{R}_M$ $\tilde{R}_S$	$(\tilde{R}_M)^{-1}$ $(\tilde{R}_S)^{-1}$ $(\tilde{R}_M)^{-1}$	$\tilde{R}_S$ $\tilde{R}_M$ $\tilde{R}_S$	$(\tilde{R}_M)^{-1}$ $(\tilde{R}_M)^{-1}$ $(\tilde{R}_S)^{-1}$	$\tilde{R}_{VS}$ $\tilde{R}_S$ $\tilde{R}_S$
$k=2$	$\tilde{R}_M$ $(\tilde{R}_E)^{-1}$ $\tilde{R}_M$	(1.1.1)	$\tilde{R}_M$ $\tilde{R}_S$ $\tilde{R}_M$	$\tilde{R}_M$ $\tilde{R}_E$ $\tilde{R}_E$	$\tilde{R}_S$ $\tilde{R}_{VS}$ $\tilde{R}_M$	$\tilde{R}_E$ $\tilde{R}_E$ $\tilde{R}_E$	$\tilde{R}_{VS}$ $\tilde{R}_{VS}$ $\tilde{R}_S$
$k=3$	$(\tilde{R}_M)^{-1}$ $(\tilde{R}_M)^{-1}$ $(\tilde{R}_S)^{-1}$	$(\tilde{R}_M)^{-1}$ $(\tilde{R}_S)^{-1}$ $(\tilde{R}_M)^{-1}$	(1.1.1)	$(\tilde{R}_M)^{-1}$ $\tilde{R}_E$ $(\tilde{R}_M)^{-1}$	$\tilde{R}_M$ $\tilde{R}_S$ $\tilde{R}_E$	$(\tilde{R}_M)^{-1}$ $(\tilde{R}_S)^{-1}$ $(\tilde{R}_M)^{-1}$	$\tilde{R}_M$ $\tilde{R}_S$ $\tilde{R}_S$
$k=4$	$\tilde{R}_M$ $\tilde{R}_S$ $\tilde{R}_M$	$(\tilde{R}_M)^{-1}$ $(\tilde{R}_E)^{-1}$ $(\tilde{R}_E)^{-1}$	$\tilde{R}_M$ $(\tilde{R}_E)^{-1}$ $\tilde{R}_M$	(1.1.1)	$\tilde{R}_M$ $\tilde{R}_S$ $\tilde{R}_{VS}$	$(\tilde{R}_M)^{-1}$ $\tilde{R}_E$ $(\tilde{R}_S)^{-1}$	$\tilde{R}_M$ $\tilde{R}_S$ $\tilde{R}_S$
$k=5$	$(\tilde{R}_S)^{-1}$ $(\tilde{R}_M)^{-1}$ $(\tilde{R}_S)^{-1}$	$(\tilde{R}_S)^{-1}$ $(\tilde{R}_{VS})^{-1}$ $(\tilde{R}_M)^{-1}$	$(\tilde{R}_M)^{-1}$ $(\tilde{R}_S)^{-1}$ $(\tilde{R}_E)^{-1}$	$(\tilde{R}_M)^{-1}$ $(\tilde{R}_S)^{-1}$ $(\tilde{R}_{VS})^{-1}$	(1.1.1)	$(\tilde{R}_M)^{-1}$ $(\tilde{R}_S)^{-1}$ $(\tilde{R}_{VS})^{-1}$	$\tilde{R}_M$ $\tilde{R}_S$ $\tilde{R}_M$
$k=6$	$\tilde{R}_M$ $\tilde{R}_M$ $\tilde{R}_S$	$(\tilde{R}_E)^{-1}$ $(\tilde{R}_E)^{-1}$ $(\tilde{R}_E)^{-1}$	$\tilde{R}_M$ $\tilde{R}_S$ $\tilde{R}_M$	$\tilde{R}_M$ $(\tilde{R}_E)^{-1}$ $\tilde{R}_S$	$\tilde{R}_M$ $\tilde{R}_S$ $(\tilde{R}_{VS})$	(1.1.1)	$\tilde{R}_{VS}$ $\tilde{R}_{VS}$ $\tilde{R}_{VS}$
$k=7$	$(\tilde{R}_{VS})^{-1}$ $(\tilde{R}_S)^{-1}$ $(\tilde{R}_S)^{-1}$	$(\tilde{R}_{VS})^{-1}$ $(\tilde{R}_{VVS})^{-1}$ $(\tilde{R}_{VS})^{-1}$	$(\tilde{R}_M)^{-1}$ $(\tilde{R}_S)^{-1}$ $(\tilde{R}_S)^{-1}$	$(\tilde{R}_M)^{-1}$ $(\tilde{R}_S)^{-1}$ $(\tilde{R}_S)^{-1}$	$(\tilde{R}_M)^{-1}$ $(\tilde{R}_S)^{-1}$ $(\tilde{R}_M)^{-1}$	$(\tilde{R}_{VS})^{-1}$ $(\tilde{R}_{VS})^{-1}$ $(\tilde{R}_{VS})^{-1}$	(1.1.1)

Table 3 The aggregated fuzzy rating of each pair of the considered criteria

	$k=1$	$k=2$	$k=3$	$k=4$	$k=5$	$k=6$	$k=7$
$k=1$	(1.1.1)	(0.22,0.56,2)	(1.5,4.33,6.5)	(0.15,0.29,0.67)	(1.5,4.33,6.5)	(0.15,0.24,0.67)	(3.5,5.67,8)
$k=2$	(0.5,2.33,4.5)	(1.1.1)	(1.5,4.33,6.5)	(1,1.67,4.5)	(1.5,5,8)	(1,1,1)	(3.5,7,9)
$k=3$	(0.15,0.29,0.67)	(0.15,0.24,0.67)	(1.1.1)	(0.22,0.56,2)	(1,3,6.5)	(0.15,0.29,0.67)	(1.5,4.33,6.5)
$k=4$	(1.5,3.67,6.5)	(0.22,0.78,1)	(0.5,2.33,4.5)	(1.1.1)	(1.5,5,8)	(0.15,0.51,1)	(1.5,4.33,6.5)
$k=5$	(0.15,0.24,0.67)	(0.125,0.23,0.67)	(0.15,0.67,1)	(0.125,0.23,0.67)	(1.1.1)	(0.125,0.23,0.67)	(1.5,3.67,6.5)
$k=6$	(1.5,4.33,6.5)	(0.5,1,1)	(1.5,3.67,6.5)	(0.5,3,6.5)	(1.5,5,8)	(1.1.1)	(3.5,6.33,8)
$k=7$	(0.125,0.18,0.29)	(0.11,0.15,0.29)	(0.15,0.24,0.67)	(0.15,0.24,0.67)	(0.15,0.29,0.67)	(0.125,0.16,0.29)	(1.1.1)

$$Bel\left(\tilde{S}_3 \geqslant \left(\tilde{S}_1, \ldots., \tilde{S}_7\right)\right) = \min(0.86,\ 0.73,\ 0.84,\ 1,\ 0.71,\ 1) = 0.71$$

$$Bel\left(\tilde{S}_4 \geqslant \left(\tilde{S}_1, \ldots, \tilde{S}_7\right)\right) = \min(1,\ 0.91,\ 1,\ 1,\ 0.88,\ 1) = 0.88$$

$$Bel\left(\tilde{S}_5 \geqslant \left(\tilde{S}_1, \ldots., \tilde{S}_7\right)\right) = \min(0.7,\ 0.56,\ 0.88,\ 0.69,\ 0.53,\ 1) = 0.53$$

$$Bel\left(\tilde{S}_6 \geqslant \left(\tilde{S}_1, \ldots., \tilde{S}_7\right)\right) = \min(1, 1, 1, 1, 1, 1) = 1$$

$$Bel\left(\tilde{S}_7 \geqslant \left(\tilde{S}_1, \ldots., \tilde{S}_6\right)\right) = \min(0.22,\ 0.125,\ 0.125,\ 0.25,\ 0.57,\ 0.12) = 0.12$$

The weight vector is represented as

$$W_p = (0.86,\ 0.97,\ 0.71,\ 0.88,\ 0.53,\ 1,\ 0.12)$$

The normalized weights vector W

$$W = (0.169,\ 0.191,\ 0.14,\ 0.174,\ 0.105,\ 0.197,\ 0.024)$$

After normalization of cardinal criteria values (Step 2 of the Algorithm) and determining the positive-ideal solution and negative-ideal solution of all considered criteria (Steps 3 and 4 of the Algorithm), the decision matrix is obtained.

Table 4 shows normalized criteria values for each supplier. The positive-ideal solution and negative-ideal solution for each considered criterion are also shown here, as in the conventional TOPSIS method.

A closeness coefficient of each suppler and rank of suppliers (Steps 6 and 7 of the Algorithm) are given in Table 5.

For certain membership function values, $\alpha = 0$, 0.5, 1 Table 5 gives argument values of fuzzy numbers which describe the closeness coefficient values of treated suppliers. The values of the closeness coefficients are used to determine the rank of treated suppliers. The rank is determined by the fuzzy number comparison method (see Appendix 3).

Table 4 Normalized criteria values: the positive-ideal and negative-ideal criteria values

	$k=1$	$k=2$	$k=3$	$k=4$	$k=5$	$k=6$	$k=7$
$s=1$	0.131579	0.121269	0.04698	(0; 0.1;0.25)	(0.75;0.9;1)	(0.35;0.5;0.65)	(0.55;0.7;0.85)
$s=2$	0.046168	0.124378	0.006711	(0.15;0.3;0.45)	(0.55;0.7;0.85)	(0.75;0.9;1)	(0.35;0.5;0.65)
$s=3$	0.084949	0.096393	0.14094	(0.35;0.5;0.65)	(0; 0.1;0.25)	(0.55;0.7;0.85)	(0.75;0.9;1)
$s=4$	0.027701	0.108209	0.100671	(0.55;0.7;0.85)	(0.15;0.3;0.45)	(0.75;0.9;1)	(0; 0.1;0.25)
$s=5$	0.184672	0.102612	0.201342	(0.75;0.9;1)	(0.35;0.5;0.65)	(0; 0.1;0.25)	(0.15;0.3;0.45)
$s=6$	0.153278	0.074627	0.302013	(0; 0.1;0.25)	(0.35;0.5;0.65)	(0.55;0.7;0.85)	(0.35;0.5;0.65)
$s=7$	0.025392	0.124378	0.006711	(0.15;0.3;0.45)	(0.75;0.9;1)	(0.35;0.5;0.65)	(0; 0.1;0.25)
$s=8$	0.193906	0.06592	0.04698	(0.35;0.5;0.65)	(0.55;0.7;0.85)	(0.15;0.3;0.45)	(0.55;0.7;0.85)
$s=9$	0.094645	0.113184	0.100671	(0.55;0.7;0.85)	(0.15;0.3;0.45)	(0.55;0.7;0.85)	(0.75;0.9;1)
$s=10$	0.05771	0.06903	0.04698	(0.75;0.9;1)	(0; 0.1;0.25)	(0.35;0.5;0.65)	(0.35;0.5;0.65)
v+	0.025392	0.06592	0.006711	(0.75;0.9;1)	(0.75;0.9;1)	(0.75;0.9;1)	(0.75;0.9;1)
v-	0.193906	0.124378	0.302013	(0; 0.1;0.25)	(0; 0.1;0.25)	(0; 0.1;0.25)	(0; 0.1;0.25)
Weight	0.169	0.191	0.14	0.174	0.105	0.197	0.024

Table 5 A closeness coefficient of each suppler and rank of suppliers

	0	0.5	1	0.5	1	Rank
$s=1$	0.195017	0.315959	0.485667	0.722221	1.13448	9
$s=2$	0.159079	0.358355	0.692592	0.841555	2.257385	2
$s=3$	0.19473	0.335779	0.546411	0.884225	1.597086	8
$s=4$	0.300239	0.482998	0.744293	0.821826	1.503852	1
$s=5$	0.35678	0.457974	0.58484	0.742183	0.957528	6
$s=6$	0.180122	0.290529	0.4466	0.684376	1.125604	10
$s=7$	0.180013	0.350421	0.627032	1.003742	1.735064	5
$s=8$	0.113914	0.274019	0.549923	0.977471	1.979534	7
$s=9$	0.225032	0.401349	0.673717	1.126836	2.2344	3
$s=10$	0.275601	0.429667	0.646985	0.945081	1.461889	4

According to the closeness coefficients, suppliers $s=4$ and $s=2$ are the most suitable ones. The firm is advised to use them as suppliers.

7 CONCLUSION

Changes occurring in business environment cause changes in a company organization and management. Many practitioners and researchers have presented the advantages of supply chain management. One of the most important problems of supply chain management is building business relationship with suppliers, which points out the common goals of managers of a firm and suppliers and underlines the fact that they use the same resources. Therefore, the problem of ranking and supplier selection is one of the most important management problems in all industrial firms existing in uncertain environment. The solution of this problem is advocated in all organizational parts of a company, which makes it the most critical factor for successful establishment of firms. The main criteria have been decided, based on the prominent research in this area carried out in Europe in 2003. The large number of criteria and possible suppliers demonstrated the complexities involved in the process of supplier selection.

In general, supplier selection problems adhere to uncertain and imprecise data which cannot be expressed by numerical values. It is appropriate to use linguistic expressions instead of numerical values for describing all uncertainties which exist in the considered problem. Modelling of linguistic variables is based on the fuzzy set theory. In this paper, all uncertainties and imprecisions present in the considered problem are modelled by triangular fuzzy numbers.

In this paper, the relative importance relation of each pair of the considered criteria is evaluated by three management team experts using linguistic variables. The weight vector is obtained by applying the extent analysis approach for the synthetic extent values of the pairwise comparison for handling fuzzy AHP. The use of fuzzy AHP does not involve cumbersome mathematical operation. The fuzzy AHP has the ability to capture the vagueness of human thinking style and effectively solve the problem of determining criteria weights in the supplier selection problem.

In practice it is known that criteria values can be crisp and uncertain. The fuzzy TOPSIS method is very flexible and can deal with the rating of both quantitative and qualitative criteria and select the suitable supplier effectively.

In this paper, an extended version of fuzzy TOPSIS is proposed, which provides more objective information for supplier selection. According to the closeness coefficient, it is possible to determine not only the ranking order but also the assessment status of all possible suppliers. All the changes, such as those in the number of criteria or their relative importance, or the number of suppliers and fuzzy number membership functions shape can be easily incorporated into the model.

Besides the aforementioned various advantages of the proposed approach for the supplier selection, this research work can be extended by adding more supplier alternatives which encompass both domestic and international suppliers as well as adding different criteria, e.g. suppliers' capacity constraints, buyers' aggregate quality, duty taxes, risk factor, etc. These extensions will undoubtedly increase computational complexities. The supplier selection decision may include environmental guidelines set by the small and medium manufacturing enterprises. Also, it should be mentioned that the proposed model presented in this paper can be easily extended to the analysis of other management decision problems in different research areas.

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NOTATION

$\tilde{c}_s$	a closeness coefficient for a supplier s ($s=1,\ldots,S$)
E	total number of decision makers
f_{sk}	-cardinal value of criterion k for supplier s, $k=1,\ldots,K'$; $s=1,\ldots,S$
f_{sk}^n	-normalized value of f_{sk}, $k=1,\ldots,K'$; $s=1,\ldots,S$
k	criterion, $k=1,\ldots,K$
K	total number of treated criteria
K'	total number of crisp criteria
s	supplier, $s=1,\ldots,S$
S^*	the best supplier
S	total number of suppliers

$\tilde{W}^e_{kk'}$ -a triangular fuzzy number $\left(x; l^e_{kk'}, m^e_{kk'}, u^e_{kk'}\right)$ describing the fuzzy rating of each decision maker

$\tilde{W}_{kk'}$ -a triangular fuzzy number $(x;\, l_{kk'}, m_{kk'}, u_{kk'})$ describing the relative importance of each pair of treated criteria, $(k=1,\ldots,K)$

w_k -the relative importance of criterion k, $(k=1,\ldots,K)$

$\tilde{v}_{sk}$ -a triangular fuzzy number $(y; l_{sk}, m_{sk}, u_{sk})$ describing the value of uncertain criterion k for a supplier s $(k=K'+1,\ldots, K;\ s=1,\ldots,S)$

v^+_k - positive-ideal value of criterion k $(k=1,\ldots,K')$

v^-_k - negative-ideal value of criterion $k\,(k=1,\ldots,K')$

$\tilde{v}^+_k$ positive-ideal value of criterion $k\,(k=K'+1,\ldots,K)$

$\tilde{v}^-_k$ -negative-ideal value of criterion $k\,(k=K'+1,\ldots,K)$

APPENDIX A: BASIC DEFINITIONS IN FUZZY SET THEORY

In this appendix, the basic definitions and notions relevant for understanding the fuzzy model in this paper are introduced [4]:

Definition A1. Fuzzy set $\tilde{A}$ is defined as a set of organized pairs:

$$\tilde{A} = \left\{x,\, \mu_{\tilde{A}}(x) \mid x \in X,\ 0 \leqslant \mu_{\tilde{A}}(x) \leqslant 1\right\} \tag{A.1}$$

where:

Fuzzy set $\tilde{A}$ is defined on the universe set $X \in R$. In general, set X can be either finite or infinite. $\mu_{\tilde{A}}(x)$ is a membership function of fuzzy set $\tilde{A}$. Each fuzzy set is completely and uniquely determined by its membership function.

Definition A2. A fuzzy number $\tilde{A}$ is a convex normalized fuzzy set $\tilde{A}$ of the real line R such that:

if exist $x_0 \in R$ such that $\mu_{\tilde{A}}(x_0)=1$

$\mu_{\tilde{A}}(x)$ is piecewise continuous.

Definition A3. Fuzzy number $\tilde{A}$ on R is to be a triangular fuzzy number if its membership function $\mu_{\tilde{A}}(x)$: $R \rightarrow [0,1]$ is equal to

$$\mu_{\tilde{A}}(x) = \left\{\begin{array}{ll} \frac{x-l}{m-l} & x \in [l,\, m] \\ \frac{x-u}{m-u} & x \in [m,\, u] \\ 0 & otherwise \end{array}\right\} \tag{A.2}$$

Where $l \leqslant m \leqslant u, l$ and u stand for the lower and upper value of the support of X respectively, and m for the modal value. The triangular fuzzy number can be denoted by (l, m, u). The support of X is the set of elements $\{x \in R |\ l < x < u\}$. When $l=m=u$, it is a non-fuzzy number by convention.

Definition A4. The α-cut of the fuzzy number $\tilde{A}$ is defined as:

$$\tilde{A}^\alpha = \left\{x,\, \mu_{\tilde{A}}(x) \middle|\, x \in X,\, \mu_{\tilde{A}}(x) \geqslant \alpha\right\} \tag{A.3}$$

Where $\alpha \in [0,1]$.

The symbol $\tilde{A}^\alpha$ represents a non-empty bounded interval contained in X, which can be denoted by $\tilde{A}^\alpha = [l^\alpha,\, u^\alpha]$, l^α and u^α are the lower and upper bounds of the closed interval, respectively.

Definition A5. A linguistic variable is a variable whose values are expressed in linguistic terms [3].

Definition A6. The operations of fuzzy numbers are based on the theorem set by Dubois and Prade [19]. Let two fuzzy numbers $\tilde{A} = \{x,\, \mu_{\tilde{A}}(x) |\, x \in R\}$, and $\tilde{B} = \{y,\, \mu_{\tilde{B}}(y) |\, y \in R\}$. The membership functions of these fuzzy numbers are montonous and subjective from zero to one and * is a continuous binary operation. Then $\tilde{A} * \tilde{B}$ is a fuzzy number which is denoted $\tilde{C} = \tilde{A} * \tilde{B}$, such as $\tilde{C} = \{z,\, \mu_{\tilde{C}}(z) |\, z \in R\}$. Values in domain of fuzzy number $\tilde{C}$, can be calculated as $z = x*y$ and

$$\mu_{\tilde{C}}(z) = \sup_{z=x*y} \min\left(\mu_{\tilde{A}}(x),\, \mu_{\tilde{B}}(y)\right) \tag{A.4}$$

Consider two triangular fuzzy numbers $\tilde{A} = (l_1,\, m_1,\, u_1)$ and $\tilde{B} = (l_2,\, m_2,\, u_2)$. Their operational laws are as follows:

1. $$(l_1,\, m_1,\, u_1) + (l_2,\, m_2,\, u_2) = (l_1 + l_2,\, m_1 + m_2,\, u_1 + u_2) \tag{A.5}$$

2. $$(l_1,\, m_1,\, u_1) - (l_2,\, m_2,\, u_2) = (l_1 - u_2,\, m_1 - m_2,\, u_1 - l_2) \tag{A.6}$$

3. $$(l_1,\, m_1,\, u_1) : (l_2,\, m_2,\, u_2) = (l_1 : u_2,\, m_1 : m_2,\, u_1 : l_2) \tag{A.7}$$

4. $$\lambda \cdot (l_1,\, m_1,\, u_1) = (\lambda \cdot l_1,\, \lambda \cdot m_1,\, \lambda \cdot u_1) \tag{A.8}$$

5. $$(\lambda,\, \lambda,\, \lambda) + (l_1,\, m_1,\, u_1) = (\lambda + l_1,\, \lambda + m_1,\, \lambda + u_1) \tag{A.9}$$

6. $$(l_1,\, m_1,\, u_1)^{-1} = \left(\frac{1}{u_1},\, \frac{1}{m_1},\, \frac{1}{l_1}\right) \tag{A.10}$$

APPENDIX B: COMPARISON OF FUZZY NUMBERS

In this Appendix a simple method of comparing fuzzy numbers and determining degree of belief that one fuzzy number is greater than or equal to one is given.

Let $\tilde{A}$ and $\tilde{B}$ be two fuzzy numbers with their supports defined on R:

$\tilde{A} = (x;\, l_1,\, m_1,\, u_1)$ and $\tilde{B} = (y;\, l_2,\, m_2,\, u_2)$

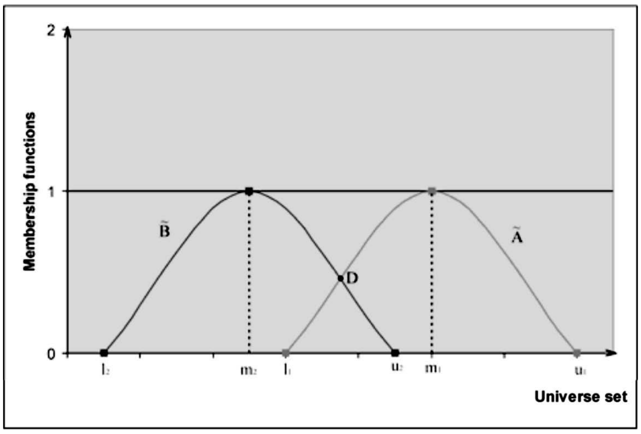

Fig. 3 Fuzzy numbers

Where l_1, l_2, u_1, u_2 are lower and upper bounds and m_1, m_2 are modal values of $\tilde{A}$ and $\tilde{B}$, respectively. Let $m_2 < m_1$ and $l_2 < l_1 < u_2$ and $l_1 < l_2 < u_1$ as in Fig. 3.

Degree of belief that $\tilde{B}$ is greater than or equal to Ã is denoted by $Bel\ (\tilde{B} \geqslant \tilde{A})$which is given using of the operation max and min [**19**]:

$$Bel\ (\tilde{B} \geqslant \tilde{A}) = \sup_{x \geqslant y} \min\left(\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(y)\right) \quad (B.1)$$

From Fig. 3 and definition (1.1) it follows that:
1.

$$Bel\ (\tilde{A} \geqslant \tilde{B}) = 1, \text{because } \mu_{\tilde{A}}(m_1) = 1 \text{ and } \mu_{\tilde{B}}(m_2) = 1 \text{ and } m_1 > m_2 \quad (B.2)$$

2. At the same time $Bel\ (\tilde{B} \geqslant \tilde{A})$ is equal to the ordinate of point D, which belongs to both $\tilde{A}$ and $\tilde{B}$, i.e. it is the supremum of intersection $\tilde{A} \cap \tilde{B}$:

$$Bel\ (\tilde{B} \geqslant \tilde{A}) = \text{ordinate of point D.} \quad (B.3)$$

When Ã and $\tilde{B}$ are triangular fuzzy numbers, the ordinate of D is given by equation (B.4):

$$Bel\ (\tilde{B} \geqslant \tilde{A}) = \frac{l_1 - u_2}{(m_2 - u_2) - (m_1 - l_1)} \quad (B.4)$$

For the full understanding of the risk analysis presented in this paper it is important to determine the degree of belief that a fuzzy number $\tilde{A}$ is bigger than/equal to K fuzzy numbers $\tilde{B}_1, ..., \tilde{B}_k, ..., \tilde{B}_K$, [**20**]:

$$Bel\ \left(\tilde{A} \geqslant \left(\tilde{B}_1, ..., \tilde{B}_k, ..., \tilde{B}_K\right)\right) = \sup_{\substack{t \geqslant t_1 \\ ..... \\ t \geqslant t_k \\ ..... \\ t \geqslant t_K}} \min\left(\mu_{\tilde{A}}(t),\ \mu_{\tilde{B}_1}(t_1), ..., \mu_{\tilde{B}_k}(t_k), ..., \mu_{\tilde{B}_K}(t_K)\right)$$

$$Bel\ \left(\left(\tilde{A} \geqslant \tilde{B}_1\right),\ and\ \left(\tilde{A} \geqslant \tilde{B}_2\right), ..., \left(\tilde{A} \geqslant \tilde{B}_k\right), ..., \left(\tilde{A} \geqslant \tilde{B}_K\right)\right) = \min_{k=1,..,K} Bel\ \left(\tilde{A} \geqslant \tilde{B}_k\right) \quad (B.5)$$